

A Time Truncated Improved Group Sampling Plans for Rayleigh and Log - Logistic Distributions

P.Kavipriya¹, A.R. Sudamani Ramaswamy²

Research Scholar, Department of Mathematics, Avinashilingam University Coimbatore, Tamil Nadu, India¹

Associate Professor, Department of Mathematics, Avinashilingam University Coimbatore, Tamil Nadu, India²

ABSTRACT : In this paper improved single and double group sampling plans based on total number of failures from the whole groups when the life time of the items follows Rayleigh or Log-Logistic distribution is proposed. The design parameters of the plans are determined such that they satisfy both producer's risk and consumer's risk simultaneously. The results are analyzed with the help of tables and examples.

KEYWORDS: Group Acceptance sampling plan, Rayleigh distribution, Log-Logistic distribution, Acceptable reliability level (ARL), and Lot tolerance reliability level(LTRL).

I. INTRODUCTION

In a competitive economy, goods stand in the market if they are of good quality. A consumer wants products of good quality at reasonable and affordable prices. Here the 'quality' can be defined in two different ways. In one sense, goods are said to be of good quality if they satisfy the consumer. In another sense, goods are said to be of good quality if they meet the expected functional use. Example for second concept, ball-bearings within the specification limits is said to be in 'control production process'. Every unit of production is tested for the standards specified. The units which do not meet the specifications are rejected. The rejected units are said to be of bad quality; as such they are not put to use. Statistical quality control is the procedure for the control of quality by the application of the theory of probability to the results of inspection of samples of the population. Sampling plans are used in the area of quality and reliability analysis. When the quality of product is related to its lifetime, it is called as life test. For a life test, variables sampling plan and attributes sampling plan are available. Acceptance sampling plans for products is one of the aspects of quality assurance in statistical quality control. Usually a single item is tested in a tester. In practice a multiple items are accommodated in a type of testers simultaneously then it is called group sampling plan. It reduces the cost and time of the experiment.

II. RELATED WORK

Aslam and Jun (2009) have designed a group acceptance sampling plan for truncated life test when the life time of an item follows either inverse rayleigh or a log- logistic distribution. Srinivasa Rao (2009) have designed a group acceptance sampling plans for the life times following a generalized exponential distribution and in (2011) he has studied the same sampling plan based on truncated life tests for the marshall-olkin extended lomax distribution. Aslam, Jun and Munir Ahmad (2009) have studied a group sampling plan based on truncated life test for a gamma distribution. In (2011) Muhammad Aslam, Chi-Hyuck Jun, Hyeseon Lee, Munir Ahmad and Mujahid Rasool have designed improved group sampling plans based on time truncated life tests for Weibull distribution.

Here in this paper improved single and double group sampling plans based on total number of failures from the whole groups when the life time of the items follows Rayleigh or Log-Logistic distribution is proposed.

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III. PROCEDURE FOR IMPROVED SINGLE GROUP SAMPLING PLAN

Here a single group sampling plan based on the total number of failures is given. As per Aslam, Jun, Lee, Ahmad and Rasool (2011) the procedure of this plan is as follows:

- (1) A random sample of size n is taken from a lot and r items are allocated to each of groups g (or testers) so that $n = rg$;
- (2) Before the termination time t_0 , r items are put on the test.
- (3) If the total number of failures from g groups is smaller than or equal to acceptance number c , lot is accepted.
- (4) If the total number of failures from g groups is greater than c before the termination time t_0 as soon as truncate the test and reject the lot.

It is also like generalization of ordinary single acceptance sampling plan. The parameters g and c are selected such that the producer's and consumer's risks both are satisfied according to given values of group sizes and true quality levels. The probability of acceptance is also calculated.

The probability of lot acceptance is given by

$$L(p) = \sum_{i=0}^c \binom{rg}{i} p^i (1-p)^{rg-i} \quad (1)$$

Where p is the probability of an item in a group fails before the termination time t_0 .

In single group sampling plan, ARL is taken as p_1 and LTRL as p_2 . The parameters g and c are selected such that the following inequalities are satisfied.

$$L(p_1) = \sum_{i=0}^c \binom{rg}{i} p_1^i (1-p_1)^{rg-i} \geq 1 - \alpha \quad (2)$$

and

$$L(p_2) = \sum_{i=0}^c \binom{rg}{i} p_2^i (1-p_2)^{rg-i} \leq \beta \quad (3)$$

It assumed that $\alpha = 0.05$ and $\beta = 0.10$. The design parameters are calculated when the life time of the items follows Rayleigh and Log-Logistic distributions for $r=5$ and $r=10$ and are presented in table 1, table 2, table 3 and table 4 respectively.

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IV. DESIGN OF SINGLE GROUP SAMPLING PLAN

RAYLEIGH DISTRIBUTION

Rayleigh (1880) derived the Rayleigh distribution to the problems in the field of acoustics. Then the Rayleigh distribution has wide applications in communication engineering and also in life testing of electro vacuum devices.

The cumulative distribution function (cdf) is given as

$$F(t; \sigma) = 1 - e^{-\frac{1}{2} \left(\frac{t}{\sigma} \right)^2}, t \geq 0 \tag{4}$$

Then the unreliability at time t_0 substituted in the above equation

$$p = F(t_0; \sigma) = 1 - e^{-\frac{1}{2} \left(\frac{t_0}{\sigma} \right)^2}$$

Where t_0 is test termination time and then t_0 is taken as $a\mu_0$ here a is constant and μ_0 is the specified life. Hence σ is taken as $\mu/\mu_0 = r_1$.

$$p = F(a\mu_0; \mu / \mu_0) = 1 - e^{-\frac{1}{2} \left(\frac{a\mu_0}{\mu/\mu_0} \right)^2}$$

$$p = F(a\mu_0; \mu / \mu_0) = 1 - e^{-\frac{1}{2} \left(\frac{a^2 \mu_0^2}{(\mu/\mu_0)^2} \right)}$$

Therefore
$$p = 1 - e^{-\frac{1}{2} \left(\frac{a^2 \pi}{(\mu/\mu_0)^2} \right)} \tag{5}$$

Where $(\mu_0^2 = \pi)$, p is the probability of an item fails before termination time t_0 .

LOG-LOGISTIC DISTRIBUTION

The cumulative distribution function (cdf) given of log-logistic distribution is given as

$$F(t, \sigma) = \frac{(t/\sigma)^\lambda}{1 + (t/\sigma)^\lambda}, t \geq 0 \tag{6}$$

It is assumed that $\lambda = 2$ and the failure probability of the item is given as

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$$p = F(a\mu_0, \mu / \mu_0) = \frac{(a\mu_0)^2}{(\mu / \mu_0)^2 + (a\mu_0)^2}$$

where $\mu_0 = 1.5708$ and

$$p = \frac{(1.5708a)^2}{\left(\frac{\mu}{\mu_0}\right)^2 + (1.5708a)^2} \tag{7}$$

Now r_1 is ARL as mean ratio at producer’s risk and r_2 is LTRL at consumer’s risk. The design parameters are selected such that they satisfy the following inequalities.

$$L\left(p_1 \mid \frac{\mu}{\mu_0} = r_1\right) \geq 1 - \alpha \tag{8}$$

$$L\left(p_2 \mid \frac{\mu}{\mu_0} = r_2\right) \leq \beta \tag{9}$$

The design parameter values and probability acceptance are determined for proposed single group sampling plan when the life time of items follows Rayleigh and Log-Logistic distributions.

Table 1 Proposed Single Group Sampling Plan for Rayleigh Distribution (r=5)

β	$\mu/\mu_0=r_1$	g	c	n=r*g	L(P ₁)							
					a=0.3	a=0.4	a=0.5	a=0.6	a=0.7	a=0.8	a=0.9	a=1.0
0.25	2	2	5	10	1.0000	1.0000	0.9999	0.9993	0.9968	0.9861	0.9701	-
	4	1	2	5	1.0000	1.0000	0.9999	0.9996	0.9990	0.9979	0.9960	0.9929
	6	1	2	5	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9993
	8	1	1	5	1.0000	0.9998	0.9996	0.9992	0.9986	0.9976	0.9963	0.9944
	10	1	1	5	1.0000	0.9999	0.9998	0.9997	0.9994	0.9990	0.9984	0.9976
0.10	2	3	7	15	1.0000	1.0000	1.0000	0.9998	0.9983	0.9917	0.9712	-
	4	2	3	10	1.0000	1.0000	0.9999	0.9997	0.9992	0.9979	0.9951	0.9899
	6	1	2	5	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9993
	8	1	2	5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999
	10	1	2	5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.05	2	4	9	20	1.0000	1.0000	1.0000	0.9999	0.9991	0.9942	0.9748	-
	4	3	5	15	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9995	0.9984
	6	2	3	10	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9994
	8	1	2	5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999
	10	1	1	5	1.0000	0.9999	0.9998	0.9997	0.9994	0.9990	0.9984	0.9976
0.01	2	5	11	25	1.0000	1.0000	1.0000	0.9992	0.9996	0.9961	0.9787	-
	4	2	5	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
	6	1	3	5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	8	1	2	5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999
	10	1	1	5	1.0000	0.9999	0.9998	0.9997	0.9994	0.9990	0.9984	0.9976

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By fixing the test termination time multiplier $a = 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9$ and 1.0 , consumer's risk $\beta = 0.25, 0.10, 0.05$ and 0.01 and mean ratios r_1 as $2, 4, 6, 8$ and 10 , the design parameters and probability of acceptance are determined by varying the number of items in a group say $r=5, r=10$ and presented in table 1, table 2, table 3 and table 4 respectively. Hyphens (-) represents the value of probability of acceptance not satisfying the condition.

Table 2 Proposed Single Group Sampling Plan for Rayleigh Distribution (r=10)

β	$\mu/\mu_0=r_1$	g	c	n=r*g	L(P ₁)							
					a=0.3	a=0.4	a=0.5	a=0.6	a=0.7	a=0.8	a=0.9	a=1.0
0.25	2	3	13	30	1.0000	1.0000	1.0000	1.0000	0.9998	0.9977	0.9823	-
	4	2	7	20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997
	6	2	4	20	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9995	0.9987
	8	1	3	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
0.10	2	3	13	30	1.0000	1.0000	1.0000	1.0000	0.9998	0.9974	0.9823	-
	4	2	6	20	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9984
	6	1	3	10	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9994
	8	1	2	10	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9985
0.05	2	2	11	20	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9979	0.9897
	4	2	9	20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	6	1	5	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	8	1	4	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.01	2	2	9	20	1.0000	1.0000	1.0000	0.9999	0.9991	0.9942	0.9748	-
	4	1	6	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	6	1	4	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	8	1	4	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	10	1	3	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

From the observation of proposed single group sampling plan for Rayleigh distribution, it is clear that the probability of acceptance value is increased for $r=10$ than $r=5$. Hyphens (-) represents the value of probability of acceptance not satisfying the condition.

Table 3 Proposed Single Group Sampling Plan for Log-Logistic Distribution (r=5)

β	$\mu/\mu_0=r_1$	g	c	n=r*g	L(P ₁)							
					a=0.3	a=0.4	a=0.5	a=0.6	a=0.7	a=0.8	a=0.9	a=1.0
0.25	2	2	5	10	1.0000	0.9999	0.9993	0.9961	0.9864	0.9640	0.9236	-
	4	1	2	5	1.0000	0.9999	0.9995	0.9987	0.9969	0.9937	0.9885	0.9807
	6	1	2	5	1.0000	1.0000	1.0000	0.9999	0.9997	0.9993	0.9987	0.9976
	8	1	1	5	0.9999	0.9996	0.9991	0.9982	0.9967	0.9945	0.9914	0.9872
	10	1	1	5	1.0000	0.9998	0.9996	0.9992	0.9986	0.9977	0.9963	0.9945
0.10	2	3	7	15	1.0000	1.0000	0.9997	0.9977	0.9890	0.9640	0.9920	-
	4	2	3	10	1.0000	0.9999	0.9997	0.9988	0.9964	0.9912	0.9816	0.9657
	6	1	2	5	1.0000	1.0000	1.0000	0.9999	0.9997	0.9993	0.9987	0.9976
	8	1	2	5	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997	0.9995
	10	1	2	5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999
0.05	2	4	9	20	1.0000	1.0000	0.9999	0.9988	0.9919	0.9672	-	-
	4	3	5	15	1.0000	1.0000	1.0000	0.9999	0.9997	0.9987	0.9962	0.9903
	6	2	3	10	1.0000	1.0000	1.0000	0.9999	0.9998	0.9995	0.9988	0.9974
	8	1	2	5	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997	0.9995
	10	1	1	5	1.0000	0.9998	0.9996	0.9992	0.9986	0.9977	0.9963	0.9945

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0.01	2	5	11	25	1.0000	1.0000	1.0000	0.9994	0.9942	0.9713	-	-
	4	2	5	10	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9993
	6	1	3	5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
	8	1	2	5	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997	0.9995
	10	1	1	5	1.0000	0.9998	0.9996	0.9992	0.9986	0.9977	0.9963	0.9945

Table 3 and table 4 follows Log-Logistic distribution for proposed single group sampling plan for $r=5$ and $r=10$, the probability of acceptance value for $r=5$ is minimum compared with $r=10$. Hyphens (-) represents the value of probability of acceptance not satisfying the condition.

Table 4 Proposed Single Group Sampling Plan for Log-Logistic Distribution (r=10)

β	$\mu/\mu_0=r_1$	g	c	n=r*g	L(P ₁)							
					a=0.3	a=0.4	a=0.5	a=0.6	a=0.7	a=0.8	a=0.9	a=1.0
0.25	2	3	13	30	1.0000	1.0000	1.0000	0.9997	0.9960	0.9753	-	-
	4	2	7	20	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9992	0.9971
	6	2	4	20	1.0000	1.0000	1.0000	0.9999	0.9996	0.9988	0.9968	0.9925
	8	1	3	10	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9997
	10	1	2	10	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9985
0.10	2	3	13	30	1.0000	1.0000	1.0000	0.9997	0.9960	0.9753	-	-
	4	2	6	20	1.0000	1.0000	1.0000	1.0000	0.9997	0.9987	0.9957	0.9880
	6	1	3	10	1.0000	1.0000	1.0000	0.9999	0.9998	0.9995	0.9988	0.9974
	8	1	2	10	1.0000	1.0000	0.9999	0.9997	0.9993	0.9985	0.9972	0.9950
0.05	2	2	11	20	1.0000	1.0000	1.0000	1.0000	0.9995	0.9970	0.9871	0.9607
	4	2	9	20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
	6	1	5	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	8	1	4	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	10	1	3	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
0.01	2	2	9	20	1.0000	1.0000	0.9999	0.9988	0.9919	0.9672	-	-
	4	1	6	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
	6	1	4	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998
	8	1	4	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
10	1	3	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	

From the above tables one can observe that as mean ratio (r_1) increases there is a decrease in number of groups (g) and acceptance number (c). At the same time there is increase in probability of acceptance when test termination time multiplier decreases. Hyphens (-) represents the value of probability of acceptance not satisfying the condition.

EXAMPLE 1

Suppose that an experimenter wants to establish that the life time of the electrical devices produced in the factory ensures that the true unknown mean life is at least 1000 hrs when the mean ratio is 2 with $\beta=0.05$ and test termination time multiplier $a=0.8$. Following are the results obtained when the life time of the item follows different life time distributions namely Rayleigh distribution and Log-Logistic distribution.

Rayleigh Distribution

Let the distribution followed be Rayleigh distribution. From the table 1 number of groups $g=2$, acceptance number $c=5$ and number of items in each group is $r=5$ and the sample size $n=r*g=10$ for the above example. The lot is accepted if the total number of failures from 2 groups are not more than 5. Probability of acceptance for improved single group sampling plan from table 1 is 0.9942. In this case one can note that the probability of acceptance is 0.9998, 0.9999, 1.0000 and 0.9990 when the ratio of mean ratio 4,6,8 and 10 respectively.

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Log-Logistic Distribution

Let the distribution followed be Log-Logistic distribution. From table 3 the number of groups is 2, acceptance number $c=5$ and number of items in each group is $r=5$ and the sample size is $n=r*g=10$ for the above example. If the total number of failures from 2 groups are not more than 5 means the lot is accepted. Probability of acceptance for improved single group sampling plan from table 3 is 0.9672. In this case one can note that the probability of acceptance is 0.9987, 0.9995, 0.9999 and 0.9977, when the mean ratio 4, 6, 8 and 10 respectively.

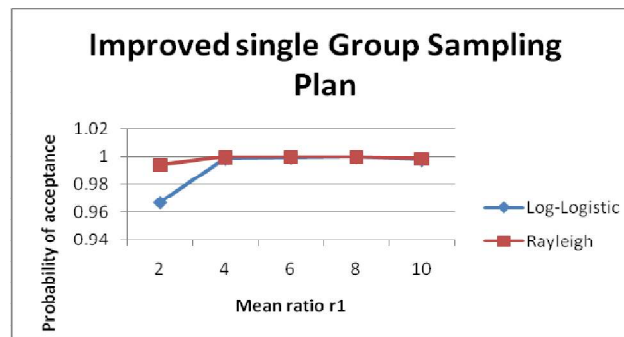


Figure 1: Rayleigh and Log-Logistic distributions for improved single group sampling plan

From the figure 1 compares the probability of acceptance when the life time of item follows Rayleigh and Log-Logistic distributions for different mean ratios. From the above figure one can conclude that the probability of acceptance for Log-Logistic distribution is less than the probability of acceptance for Rayleigh distribution only for mean ratio $r_1=2$. But for the other values, the probability of acceptance for both the distribution are approximately equal.

V. OPERATING PROCEDURE OF THE IMPROVED DOUBLE GROUP SAMPLING PLAN

The following is the procedure presented by Aslam M, Jun C-H, Lee H, Ahmad M and Rasool M (2011).

FIRST STAGE

- (i) Draw the first random sample of size n_1 from a lot;
- (ii) Allocate r items to each of g_1 groups (or testers) so that $n_1=r*g_1$.
- (iii) Put r items are put on the test before termination time t_0 ;
- (iv) Accept the lot if the total number of failures from g_1 groups is smaller than or equal to c_{1a} ; and
- (v) Truncate the test and reject the lot as soon as the total number of failures is greater than or equal to $c_{1r}(> c_{1a})$ before t_0 . Otherwise go to the second stage.

SECOND STAGE

- (i) Draw the second random sample of size n_2 from a lot;
- (ii) Allocate r items to each of g_2 groups so that $n_2 = r*g_2$;
- (iii) Put r items on test before the termination time t_0 ;

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- (iv) Accept the lot if total number of failures from g_1 and g_2 groups is smaller than or equal to $c_{2a}(\geq c_{1a})$. Otherwise.
- (v) Truncate the test and reject the lot.

Therefore the above given double group sampling plan the five parameters are g_1, g_2, c_{1a}, c_{1r} and c_{2a} . This plan is different from ordinary double acceptance sampling plan. When $c_{1r} = c_{1a} + 1$, then x_1 is denoted as total number of failures from g_1 groups and parameters n_1 and p use to follow binomial distribution. We know that p is failure probability at termination time t_0 . Now the lot acceptance probability at first stage is given by

$$P_a^{(1)} = P(X_1 \leq c_{1a}) = \sum_{j=0}^{c_{1a}} \binom{n_1}{j} p^j (1-p)^{n_1-j} \quad (10)$$

Then the lot rejection probability at first stage as

$$P_r^{(1)} = \sum_{j=c_{1r}}^{n_1} \binom{n_1}{j} p^j (1-p)^{n_1-j} \quad (11)$$

Now the lot is accepted from the second stage if the decision not taken from the first stage and the total number of failures from g_1 and g_2 groups is denoted by X_2 is smaller than or equal to c_{2a} .

$$P_a^{(2)} = P(c_{1a} + 1 \leq X_1 \leq c_{1r} - 1, X_1 + X_2 \leq c_{2a})$$

$$= \sum_{x=c_{1a}+1}^{c_{1r}-1} \binom{n_1}{x} p^x (1-p)^{n_1-x} \left[\sum_{i=0}^{c_{2a}-x} \binom{n_2}{i} p^i (1-p)^{n_2-i} \right] \quad (12)$$

Hence the probability of lot acceptance is given by

$$L(p) = P_a^{(1)} + P_a^{(2)} \quad (13)$$

VI. DESIGN OF PROPOSED DOUBLE GROUP SAMPLING PLAN

Aslam M, Jun C.H, Lee H, Ahmad M and Rasool M (2011) designed the proposed double group sampling plan and applied Weibull distribution. Fixing these parameters $g_1, g_2, c_{1a}, c_{2a}, c_{1r}$, consumers risk and mean ratios, the probability of acceptance and ASN value is determined. Let the failure probabilities be p_1 and p_2 corresponding to consumer's risk and producer's risk respectively. Consider ASN under LTRL, p_2 is given by

$$ASN(p_2) = rg_1 + rg_2(1 - P_a^{(1)} - P_r^{(1)})$$

Then the optimization problem is considered as

$$\text{Minimize } ASN(p_2) = rg_1 + rg_2(1 - P_a^{(1)} - P_r^{(1)})$$

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Subject to

$$L(p_1) \geq 1 - \alpha$$

and

$$L(p_2) \leq \beta$$

In this paper for the above double group sampling plan the author applies Rayleigh and Log-Logistic distributions.

Table 5 Proposed Double Group Sampling Plan for Rayleigh Distribution(a=0.3)

r=5						r=10							
β	$\mu/\mu_{0=r_1}$	c_{1r}	c_{2a}	g_1	g_2	ASN	$L(p_1)$	c_{1r}	c_{2a}	g_1	g_2	ASN	$L(p_1)$
0.25	2	3	3	2	2	12.2	0.9946	4	5	2	2	30.3	0.9952
	4	2	2	2	2	10.8	0.9964	2	2	1	1	10.8	0.9964
	6	2	1	1	1	5.0	0.9995	2	2	1	1	10.3	0.9993
	8	2	1	1	1	5.0	0.9998	2	2	1	1	10.2	0.9998
0.10	2	5	4	3	2	19.1	0.9998	5	6	3	2	42.9	0.9965
	4	3	2	2	1	10.4	0.9999	2	3	1	1	10.8	0.9967
	6	2	2	2	1	10.1	0.9993	2	2	1	1	10.3	0.9993
	8	2	2	1	1	5.0	1.0000	2	2	1	1	10.2	0.9998
0.05	2	5	5	4	2	25.0	0.9995	5	7	3	3	49.4	0.9965
	4	3	3	3	2	16.2	0.9997	3	2	2	1	21.6	0.9988
	6	3	3	2	2	10.3	1.0000	3	2	2	1	20.7	0.9999
	8	3	2	1	1	5.0	1.0000	3	1	2	1	20.4	0.9990
0.01	2	6	7	5	3	33.7	0.9998	7	8	5	3	74.8	0.9984
	4	3	2	3	3	16.8	0.9988	4	4	3	2	34.6	0.9999
	6	2	1	2	2	10.3	0.9979	3	2	3	1	31.1	0.9997
	8	2	1	2	1	10.1	0.9995	2	2	2	1	20.4	0.9991

The parameter values mean ratio ($=2, 4, 6, 8$), consumer's risk ($= 0.25, 0.10, 0.05, 0.01$), test termination time multiplier ($a=0.3$) and c_{1a} ($=0$) are fixed for table 5 and table 6 respectively where the number of items in a group is varied as $r=5$ and $r=10$ to determine value for the probability of acceptance and average sample number using Rayleigh and Log-Logistic distributions.

Table 6 Proposed Double Group Sampling Plan for Log-Logistic Distribution(a=0.3)

r=5						r=10							
β	$\mu/\mu_{0=r_1}$	c_{1r}	c_{2a}	g_1	g_2	ASN	$L(p_1)$	c_{1r}	c_{2a}	g_1	g_2	ASN	$L(p_1)$
0.25	2	3	3	2	2	14.0	0.9815	4	5	2	2	32.8	0.9791
	4	2	2	2	2	11.2	0.9912	2	2	1	1	11.2	0.9912
	6	2	1	1	1	5.1	0.9987	2	2	1	1	10.5	0.9983
	8	2	1	1	1	5.0	0.9996	2	2	1	1	10.3	0.9995
0.10	2	5	4	3	2	20.5	0.9985	5	6	3	2	45.6	0.9806
	4	3	2	2	1	10.6	0.9995	2	3	1	1	11.2	0.9921
	6	2	2	2	1	10.2	0.9983	2	2	1	1	10.5	0.9983
	8	2	2	1	1	5.0	0.9999	2	2	1	1	10.3	0.9995
0.05	2	5	5	4	2	26.5	0.9967	5	7	3	3	53.4	0.9803
	4	3	3	3	2	16.8	0.9989	3	2	2	1	22.3	0.9957
	6	3	3	2	2	10.5	1.0000	3	2	2	1	21.1	0.9996

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	8	3	2	1	1	5.0	1.0000	3	1	2	1	20.6	0.9977
0.01	2	6	7	5	3	36.0	0.9984	7	8	5	3	77.5	0.9847
	4	3	2	3	3	17.7	0.9957	4	4	3	2	36.7	0.9992
	6	2	1	2	2	10.5	0.9949	3	2	3	1	31.6	0.9989
	8	2	1	2	1	10.1	0.9989	2	2	2	1	20.6	0.9978

The probability of acceptance and ASN values are calculated for g_1, g_2, c_{1r}, c_{2a} for Rayleigh and Log-Logistic distributions with $r=5$ and $r=10$ and presented in table 5 and table 6 respectively. ASN value is almost same. From the above tables, one can conclude that the ASN value of Rayleigh distribution is minimum compared with Log-Logistic distribution.

VII. ASN VALUES OF RAYLEIGH AND LOG-LOGISTIC DISTRIBUTIONS ARE COMPARED

In table 7 compares the Average sample number values of Rayleigh and Log-Logistic distributions with ASN of Weibull distribution ($m=2$) as per Aslam M, Jun C.H, Lee H, Ahmad M and Rasool M (2011) . ASN value is minimized using optimization problem for double group sampling plan.

Table 7 ASN Values with Rayleigh and Log-Logistic Distributions for $\beta=0.05$

Me an Ratio r_1	r =10			r=5		
	Log-logistic distribution	Rayleigh distribution	Weibull distribution m=2	Log-logistic distribution	Rayleigh distribution	Weibull distribution m=2
2	53.4	49.4	52.4	26.5	25.0	46.5
4	22.3	21.6	20.9	16.8	16.2	20.9
6	21.1	20.7	20.9	10.5	10.3	20.9
8	20.6	5.0	20.9	20	5.0	20.9

In the above table with the value of mean ratio ($= 2,4,6,8$), the number of items in a group ($r=5, r=10$) and consumer's risk $\beta=0.05$. It is observed that ASN values of three distribution values decreases when mean ratio value increases.

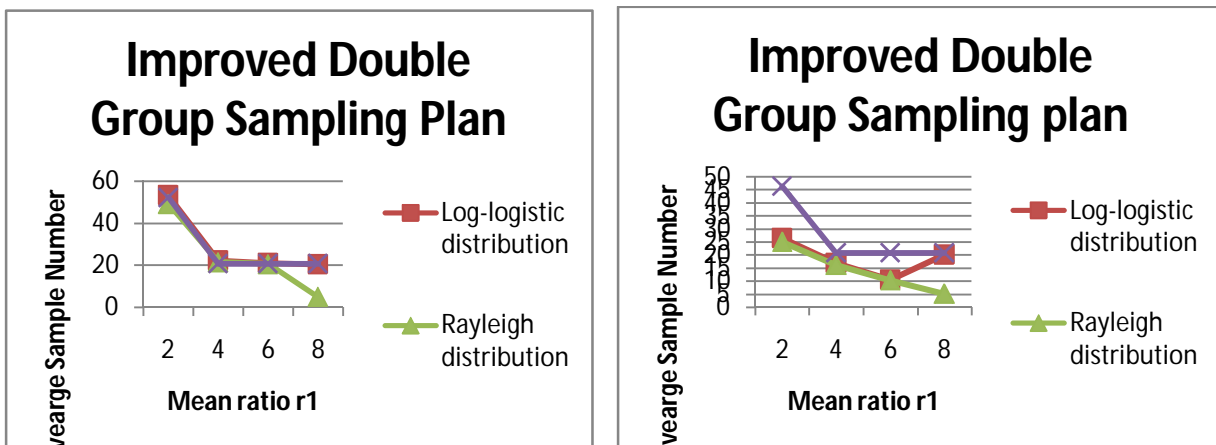


Figure 2: Comparison of Rayleigh, Log-Logistic and Weibull distributions for $r=10$ (left) and $r=5$ (right)

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From the analyses of table 7 and figure 2, ASN value of Rayleigh distribution is minimum as compared along with Weibull ($m=2$) and Log-Logistic distributions.

VIII. CONCLUSION

One can concluded that if the life time of the item follows Rayleigh distribution the probability of acceptance value is greater than the probability of acceptance of Log-Logistic distribution for proposed single and double group sampling plans. One can observe that as mean ratio increases there is decrease in number of groups and acceptance number. And at the same time there is increase in probability of acceptance when test termination time multiplier decreases. ASN value for Rayleigh distribution is minimum while comparing with Weibull ($m=2$) and Log-Logistic distributions.

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